The Truth About Financial Economics, Pensions, and Fair Value
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Accounting standards boards in the US and internationally are tasked with creating standards on how to measure the fair value of pensions. Unfortunately, a significant roadblock stands in their way; a fundamental error of financial economics that has misled the accountants to apply what I call the 'funding irrelevancy rule'. To reach their goal, it will be necessary to recognize and correct this error, and then incorporate methods that go beyond the traditional actuarial model.

The Fundamental Error

Financial economics has failed to recognize that many FAS 157 Level 3 instruments – including pension liabilities - are not subject to arbitrage, and are thus not subject to the law of one price. (The law of one price states that two traded instruments with the same cash flows should be priced the same.) The reason pensions are not subject to arbitrage: a prerequisite for the possibility of arbitrage is a market with easy buying and selling at set prices.

FAS 157 is the Financial Accounting Standards Board (FASB) statement defining fair value. Level 3 instruments are those that lack 'observable inputs'; they do not have easily observable market prices. In contrast, Level 1 instruments (e.g., stocks and bonds) do have easily observable market prices. The International Accounting Standards Board (IASB) fair value draft uses the same three level hierarchy.

Consider two traded financial instruments with identical cash flows, one priced at $100 and another priced at $110. An arbitrageur can make a quick profit by following these steps, which are the blueprint for nearly pure arbitrage:

1. Recognize the mispricing available in the marketplace.

2. Transact for the mispriced instrument such that the arbitrageur underpays or gets overpaid.

3. Conduct the opposite transaction for the other instrument(s) involved at the correct market price. (At this point, the arbitrageur has bought the $100 asset and sold short the $110 asset, regardless of the correct pricing of the assets.)

4. Exit the positions when the prices of the two instruments have converged (or immediately for pure arbitrage). (Arbitrage theory suggests that almost all the time, the prices of two instruments with identical cash flows will quickly converge.)

Now, say there are 2 identical frozen pension plans seeking to settle their liabilities by transferring them to another company. Your firm is interested in a pension buyout transaction, and has the census data for both plans. Plan Sponsor A is offering $100 million for the settlement of their liabilities and Plan Sponsor B is offering $110 million.

From the financial economics perspective, there's clearly some mispricing here – identical sets of cash flows with two different asking prices. Can anyone make a first-order (i.e., not considering second or higher order effects such as tax) nearly pure arbitrage profit in this situation? Upon reflection, you should agree that no, it is not possible to make a nearly pure arbitrage profit from this situation. What prevents this possibility is the absence of a market with easy buying and selling at set prices.

In particular, it is impossible for a third party arbitrageur to undertake both transactions in steps 2 and 3. He can accept the higher-priced $110 million settlement amount from Plan Sponsor B, but there is no analog of 'short selling' available to him to transact a 'reverse settlement' with Plan Sponsor A.

For both Plan Sponsors A and B, there's no assurance that they will be able to find a counterparty agreeable to their offer price to settle the liability. There is an inevitable risk that either step 2 or 3 will not happen, so it is impossible for Plan Sponsors A and B to act as arbitrageurs as well.

I used pension liabilities as an example, but the same argument can applied to any FAS 157 Level 3 instrument that is not a synthetic composite of Level 1 instruments. (In the 'Level 1 composite' case, arbitrage is possible and the law of one price does apply.)

Conclusion: The law of one price does not apply to pension liabilities and other non-arbitrageable FAS 157 Level 3 instruments. The law of one price is exclusively derived from no-arbitrage arguments. There is no other reason to believe that it applies to pension liabilities.

Note that the question of whether an instrument is arbitrageable is different from whether it's traded. For example, real estate (i.e., individual land plots and buildings) is an example of a non-arbitrageable class of FAS 157 Level 2 instruments, notwithstanding an active market for real estate. Real estate instruments are non-arbitrageable because they are heterogeous, and each sale is individually negotiated (there is no easy buying and selling at set prices.) And, you can't 'short' a building, as you can short a traded security. Real estate is subject to flipping, which differs from arbitrage in that it requires risk-taking.

Principles other than the law of one price, however, do extend to pension liabilities; in particular, rational choice. Pension liabilities funded with risky assets are associated with a concept known as a Bader swap[1]. A Bader swap is a theoretical derivative used by financial economists to illustrate the law of one price. Instead of thinking of the pension plan assets as a risky portfolio (RP) with value $V, imagine the pension assets consist of $V of risk-free bonds, plus a Bader swap. The Bader swap consists of a $V short position in risk-free bonds, and the portfolio RP with value $V.

The Bader swap per se has a fair value of zero. However, when used to fund a non-arbitrageable liability, the Bader swap affects the plan sponsor's exit price of that liability, and thus its fair value. The Bader swap represents a quantity I call PVEG - present value of (ex post) expected gains. PVEG is a legitimate expectation resulting from an expected return on risky assets higher than the risk-free rate. Because a pension plan sponsor is rational, he would require compensation for giving up PVEG (provided that he expects to gain a financial advantage from PVEG.) He is free to place a value on PVEG because pension liabilities are not subject to arbitrage. We conclude that the plan sponsor's exit price would include an adjustment for PVEG. By the same token, his exit price would also reflect the risk of poor financial performance inherent in risky investments, as well as all other risks associated with the pension liability.

Reasonable Assumptions

The pro-forma traditional actuarial valuation model (i.e., before specifying the assumptions) has the virtue of producing a single answer for the plan's liability. A single answer (as opposed to a range) is a requirement for pension accounting, as this number must plug into the plan sponsor's balance sheet. So, as a final step, the pro-forma traditional actuarial model is optimal. The question is how to get reasonable assumptions to be used in this final step.

First, it is essential to recognize that significant parameters that affect the cost of a pension plan are best thought of as random variables. Also, these parameters are not limited to just investment return. For example, salary scale and retirement rates can have a significant impact on costs. Because these parameters extend in time, their effect is geometric; hence the lognormal distribution is a natural choice to model them. This paper will present a model that factors the stochastic experience risk of all significant parameters into the discount rate.

Fair Value Definition

The following definition of fair value captures the concepts presented in this paper:
The fair value of an instrument is a unique rational and unbiased estimate of the equilibrium sale price of the instrument between market participants with average utility with respect to accompanying risks and opportunities. (Average utility is taken among market participants for that instrument.)

Like the FAS 157 definition, this definition envisions a hypothetical transaction wherein the plan sponsor pays a **plan receiver** to assume the pension liability. In order to capture all the accompanying risks and opportunities, the hypothetical transaction here is for the entire pension plan, including future benefit accruals for existing and new entrants. The transaction is paid for by the transfer of plan assets, plus financing in the form of the former plan sponsor continuing to make the same contribution to the plan they would have made if the sale had not occurred in accordance with an established credible funding policy. Thus, the plan receiver has access to plan assets at the same time that the plan sponsor would have, in the absence of a sale.

The object of considering this hypothetical transaction is to determine the equilibrium sale price represented by this transaction, and the discount rate inherent in that price. The equilibrium sales price is conceptually the same as what IASB calls exit price (not what FASB calls exit price, which may be closer in concept to IASB's 'exit value'). As it's been pointed out, the exit price of a market participant with average utility is also conceptually the same as the entry price. The exit price is defined as follows:

-for an asset, the minimum price the asset holder would sell the asset for (or, the greatest available liquidation price if greater).
-for a liability, the maximum price the liability holder would pay to settle it (or, the lowest available liquidation price if lower).

The above definition of fair value differs from both FASB's and IASB's concepts of fair value by including opportunities to be factored into an exit price. In particular, the expectation of returns in excess of a risk-free rate is an opportunity associated with risky pension assets (funding a non-arbitrageable liability) that is considered by this definition. This definition represents a departure, therefore, from the 'funding irrelevancy rule' found in IAS 19 and FAS 87; i.e., that the value of the pension plan's liability is independent of whether the plan is funded or how plan assets are invested. The funding irrelevancy rule follows from the law of one price; however, I've just shown that the law of one price does not apply to pension liabilities.

The Funding Irrelevancy Rule

Arguments in favor of the funding irrelevancy rule are addressed here:

A liability is a liability, independent of funding considerations. True enough. However, a liability per se is different from the fair value of that liability. A liability is the obligation to perform a specified set of actions. The fair value of any instrument is an epistemological function of that instrument; it resides in the minds of market participants for that instrument. It is not an ontological property of the instrument. It would be irrational for a market participant to fail to consider the risks and opportunities associated with the investment of plan assets funding the liability.

To illustrate that fair value is an epistemological function, consider an ounce of gold. The fair value of an ounce of gold is the value that market participants collectively place on it. It is not an intrinsic property of the gold itself. If all humans suddenly vanished from the planet, but the gold remained, the gold would no longer have a fair value. This is because fair value resides in the minds of market participants.
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The pension liability discount rate must be independent of plan assets to ensure comparability among pension plan sponsors. This is false. The most meaningful measure of a plan sponsor's aggregate liability (i.e., before attribution to time periods) is the risk-adjusted estimate of the plan sponsor's present value of future contributions. Indeed, all of the pension plan sponsor's exposure comes in the form of future plan contributions. By

ignoring the plan sponsor's investment opportunities and risks affecting future contribution levels, current accounting rules enforce a false comparability.

Under FASB Concepts Statement 7, the default time value of money is a risk-free rate, in accordance with established financial economic theory. For FAS 157 Level 1, this result follows directly from the Fundamental Theorem of Arbitrage-Free Pricing (Harrison & Pliska). While I don't claim an encyclopedic knowledge, I am confident that there is no corresponding theory to support this conclusion for FAS 157 Level 3 instruments. Indeed, how is it possible to gather empirical evidence about prices of instruments that are unobservable? However, in the model that follows, plan sponsor contributions (rather than benefit payments) are discounted at a risk-free, or nearly risk-free rate.

Pension accounting should reflect only earned risk premiums (in the market value of assets), not unearned risk premiums (in the liability). This is essentially a corollary to the funding irrelevancy rule. As there is no theoretical support for the funding irrelevancy rule, what's left is to examine the empirical evidence. At least in the US, pension plan sponsors invest in risky assets and are unwilling to purchase annuities to settle pension liabilities at prices prevailing over the last decade or so. It appears that they do reflect unearned risk premiums in their exit price.

In the UK, there has been a fair amount of settlements in the form of pension buyouts, even though they are priced about 10% - 30% above the IAS 19 liability measure[2]. The UK is a special case for several reasons. First, pensions must be inflation-indexed, leading to inflation and longevity risks unique to the UK. Second, plan investment decisions are made by a pension board, not the sponsor. For plan sponsors whose pension board has put them in gilts and other low risk – low return investments, a pension buyout can be a relatively attractive option. Finally, there is a (self-reinforcing) sense of dread surrounding all aspects of UK pensions; fear is indisputably a factor in plan sponsors' exit prices. Nonetheless, the pension buyout price should be regarded as a ceiling to the pension liability rather than a 'market price', because the percentage of pension liabilities being settled via buyout is still small.

To illustrate that IASB is taking the wrong course toward fair value, consider IASB's "Preliminary Views on Insurance Contracts". Insurance contracts are non-arbitrageable Level 3 instruments. Therefore, as discussed above, there is no justification for applying the law of one price to determine the discount rate for future cash flows. Nonetheless, this is precisely the approach IASB takes. This discount rate (based on the law of one price) is used to determine what IASB calls 'current exit value', which is deemed to be indistinguishable from the fair value of insurance contract liabilities.

Note that 'current exit value' is based neither on empirical evidence nor on a model predicting the maximum settlement price insurance companies would pay to settle the liabilities. IASB has abandoned the exchange price notion in this case, which is the true essence of fair value. Instead, the concept of 'exit price' is discarded in favor of an imposed 'exit value'. And, the 'wisdom of the crowd' of insurance companies is replaced by the opinion of a select group of valuation experts. Ironically, the class of instruments whose fair value the experts are consulted on is the class of non-arbitrageable Level 3 instruments, to which they incorrectly and unjustifiably apply the law of one price.

Informational Cascade

How did this error (unjustifiably applying the law of one price to non-arbitrageable instruments) become so prevalent? How is it that a likely majority of economists, as well as many actuaries and accountants are simply wrong on this key point?

Although I cannot fully account for this error, the genesis and history of this error has all the earmarks of a textbook informational cascade. In an informational cascade, decision-makers ignore their private signals and instead make their choice based on the overwhelming number of 'votes' previously made by others. Ironically, the history of this error is closely related to some of the more spectacular successes of financial economics, namely the Modigliani-Miller theorem, the Black-Scholes option pricing formula, and the Fundamental Theorem of Arbitrage-Free Pricing. These results form the basis of modern financial engineering.

What went unrecognized as these successes were occurring is the fact that the instruments that these results apply to are exclusively FAS 157 Level 1. (For one thing, this FAS 157 vocabulary didn't exist until 2006.) Meanwhile, IAS 19 and FAS 87 established the funding irrelevancy rule for pension accounting, and eventually a single standard (AA bond rates) for discount rates. Then, a movement to discard the traditional pension model in favor of a financial economics model took hold, first in the UK, and now in the US. Most recently, the fair value accounting movement took hold, and fair value accounting procedures are now in the process of being formulated.

Along the way, some questioned the application of the law of one price to non-arbitrageable liabilities as running counter to common sense[3]. Financial economists have labeled pension plan sponsors and their advisors who followed their own common sense as irrational, and possibly greedy, lazy, or stupid[4]. Perhaps seduced by the beauty of the Modigliani-Miller no arbitrage arguments, financial economics proponents believe (or 'know') that arbitrage is always possible, even for Level 3 instruments. Dmitri Mindlin may have been the first to point out that pension liabilities are not arbitrageable[5], and finally, this paper says why: like other FAS 157 Level 3 liabilities, they lack easy trading with set prices.

Level 1 Financial Economics vs. Level 3 Financial Economics

Proponents of current financial economics may prefer to gloss over the following distinction, but it is instructive to contrast Level 1 financial economics (i.e., financial economics as it applies to FAS 157 Level 1 instruments) and Level 3 financial economics.The former is a science with a strong mathematical foundation, while the latter is a belief system.

Level 1 financial economics is a science with a robust mathematical model. This model is exemplified by Harrison & Pliska's[6] Fundamental Theorem of Arbitrage-Free Pricing, which is based on a frictionless market with continuous trading. This theorem represents the culmination of several preceding notable results, including the Modigliani-Miller theorem and the Black-Scholes option pricing formula. The continuous trading model is an excellent fit for FAS 157 Level 1.

Turning to Level 3 financial economics, we pass from the sublime to the ridiculous. Level 3 instruments do not fit into the Harrison & Pliska model: Level 3 instruments are scarcely traded if at all (not continuously traded), and Level 3 instruments lack an observable price process and a filtration under which securities' prices are revealed to market participants as they change over time. The model cannot be directly applied to a Level 3 instrument.

Level 3 financial economics proceeds beyond this point by committing the fallacy of sweeping generalization. An example of this fallacy: All birds have wings; all birds are animals; therefore, all animals have wings. This particular formation of the fallacy is: Level 1 instruments are known to be subject to the Law of One Price; All Level 1 instruments are instruments; therefore, all instruments (including Level 3) are subject to the Law of One Price. Only after committing this fallacy can you posit the existence of a reference security with matching cash flows to place a value on the Level 3 instrument.

A Corrected Model

Described below is a model intended to derive a discount rate to calculate the fair value of a pension liability. It's a universal discount rate model, applicable to all DB plans, US & international, pension & postretirement, private sector & public sector, single-employer & multiemployer, funded & unfunded.

One new term is introduced by the model: present value of future (sponsor) contributions, or PVFC. PVFC is a simplified risk-adjusted calculation of a discounted set of projected sponsor contributions to the plan. The model also uses MVA (market value of assets) and PVFB (present value of future benefits).

The model starts with the equation PVFC + MVA $\geq$ PVFB. Conceptually, all plan benefits must come from current plan assets, investment return on plan assets, or from future contributions, although it is possible for the plan to become overfunded. The model changes this inequality to the equality PVFC + MVA = PVFB in order to estimate the pension plan sponsor's maximum exit price.

The discount rate produced by the model is the discount rate that, when used to calculate PVFB, makes the equation true. The rationale behind this method is that if this discount rate is used for the settlement, the plan sponsor's PVFC remains unchanged; therefore he should be indifferent to settling at this rate. To see why, say the sponsor wished to settle the entire PVFB. He would be willing to pay MVA + PVFC; his immediate out-of pocket cost is PVFC, so PVFC hasn't changed.

Say he wanted to settle a fraction k of the PVFB, leaving (1-k) * PVFB remaining. He would be willing to pay k *(MVA + PVFC). Say he pays the entire k*(MVA + PVFC) from plan assets for the settlement, and PVFC' is his new present value of contributions after the settlement. You have (1-k)*MVA – k*PVFC + PVFC' = (1-k)*PVFB. Multiplying the basic equation by (1-k), you also have (1-k)* MVA + (1-k)*PVFC = (1-k)*PVFB. Solving, PVFC' = (1-k)*PVFC + k*PVFC = PVFC, so PVFC is unchanged. A similar argument works if part or all of the settlement is paid outside the plan.

Most of the work involved in this model is in calculating PVFC. The calculation of PVFC starts with a regulatory framework or credible funding policy used to determine the plan's contributions. In this example, say the plan is a US DB plan subject to PPA 2006, and the funding policy is to make the minimum required contribution.

Then, all demographic and economic parameters (such as salary scale) that significantly impact the plan's future payouts are identified. Say, in this case, salary scale and retirement rates are identified as the parameters that are to be risk-adjusted. Say there are p such parameters, so we have p = 2.

Each parameter (e.g., age 40 salary scale) is assigned an arithmetic mean and an arithmetic variance. This variance assigned is based on experience of the entire plan, rather than individual members. For example, your assumed (arithmetic mean) age 40 salary scale might be 3.5%, and you estimate that the (arithmetic) standard deviation of this parameter for the plan as a whole is 1.0%.

The lognormal distribution is assigned, with successive years independent and identically distributed. You then solve for lognormal parameters μ and σ, and for geometric mean e^{μ} and geometric standard deviation e^{σ}. For example, for age 40 salary scale parameter X, you will have $\mu \approx 0.034355$, $1 + \mu < e^{\mu} < E(X) = 1.035$, $\sigma \approx 0.009662$ and $1 + \sigma < e^{\sigma} < 1 + \sqrt{var(X)} = 1.01$. (X is the ratio of the prospective year's pay to the preceding year's pay at age 40.) Observe that $Y = \ln(X)$ is normally distributed with mean μ and standard deviation σ.

At this point there are two equivalent possible approaches. One approach is to use stochastic forecasting, allowing future experience of $(Y - \mu)/\sigma$ through each forecast valuation date to randomly take on values of Φ from the interval [0,1]. At each forecast valuation date, the target liability (TL) and target liability normal cost (TLNC) payout streams are calculated using the arithmetic average assumed parameters. (These payout streams can then be discounted back to the forecast valuation date at different projected segment rate sets to derive TL and TLNC.) Combining these results with a projected market value of assets that has also been stochastically projected allows you to compute the forecast minimum required contribution for that trial.

An alternative **constant-Φ** approach described in this paper offers the potential advantage of requiring less processing time than stochastic processing, but still reflecting the full range of possible outcomes for each parameter. Choosing an integer n (n = 5 in our case), you calculate n equally probability-spaced representative constant-Φ arrays; i.e., for each array, for each parameter, at each time t, the cumulative distribution function of the cumulative product $\Pi_t(Y - \mu)/\sigma$ is a constant $\Phi_i = (2i - 1)/2n$, for i = 1 to n. As n increases, the arithmetic average of the n cumulative products approaches the expected cumulative product using the arithmetic mean.

For example, for $\Phi_1 = 0.1$, the first two years of experience for the age 40 salary scale are 1.022216 and 1.029657. For $\Phi_5 = 0.9$, the first two years of experience for the age 40 salary scale are 1.047846 and 1.040273. The arithmetic averages of the first two years' salary scale experience for Φ_1 through Φ_5 are 1.034989 and 1.034958.

The next step is to calculate n^p TL and TLNC payout forecast tetrahedral arrays by cohort, forecast valuation date and payout year. Each forecast will use its constant-Φ experience arrays from time zero through each forecast valuation date, and the regular arithmetic mean assumptions thereafter. The forecasts will assume new entrants (e.g. using a constant active population), and payout calculation will be truncated at a specified future date (say t = 75). In our case, we will have 25 TL and TLNC payout forecast tetrahedral arrays.

In addition, we will calculate one more set of TL and TLNC payout forecast arrays based on the arithmetic mean assumptions for all years. We will set this aside to be used as the basis for PVFB.

We will combine our 25 equally weighted TL and TLNC payout arrays with a set of five (or more) constant-Φ segment rate sets, and five (or more) constant-Φ asset return experience arrays. Please note the following important points about the asset return experience arrays:

- The arithmetic mean and variance of the asset return variable would be based on an investment allocation.
    - For the equity component of that investment allocation, the expected return would be derived from the Capital Asset Pricing Method (CAPM), with an adjustment for the term R_m in the CAPM formula for the current average market P/E ratio vs. the historical average.
    - For example, if the current market P/E ratio is 20% above the historical average benchmark, R_m could be reduced by 184 basis points for each forecast year 1 through 10 to reflect an expected return to the historical average P/E ratio.
- The investment allocation used to determine the asset return arithmetic mean and variance is the FAS 87 universe average investment allocation, or, if less risky, the plan sponsor's investment allocation. The rationale is to reflect the average utility of a market participant, unless special circumstances pertaining only to a subset of market participants apply. For example, there are a number of reasons why the plan sponsor may have de-risked his portfolio: because of overfunding, short liability duration, or an expectation that the plan will terminate soon.

We then calculate forecasted contributions, and an average is taken with the scenarios equally weighted if no parameter correlation is assumed, or with weighting based on the assumed correlations. Selected forecasts with interpolation could be used to reduce the number of computations needed. Reflected in these forecasted contributions are a credible funding policy and an assumption that estimated expenses will be paid by the plan; these are reflected in the asset rollforward.

Finally, these projected risk-adjusted contribution averages are discounted at a senior discount rate reflective of the fact that they are similar to compensation. Thus, this discount rate would be a risk-free or nearly risk-free rate, reflecting only the possibility of the plan sponsor going completely out of business. The result is PVFC. The flowchart below illustrates the entire model.

Returning to the equation MVA + PVFC = PVFB, we make a final adjustment to the model. Given the opportunity to reduce the plan sponsor's risk by settling pension liabilities without increasing PVFC, would a rational plan sponsor do it? Actually, this is a no-brainer; of course they would. In fact, a rational plan sponsor would be willing to pay something more in exchange for the reduction in risk. So, we modify the equation for the model to MVA + PVFC = PVFB/(1 + RL), where RL is a risk loading factor. RL can also be thought of as the plan receiver's expected profit margin.

Applying the model to a few specific types of plans:

- A US unfunded pension plan would have a PVFC (representing risk-adjusted future benefit payments) discounted at a nearly risk-free rate. In this case, the model would likely produce a discount rate lower than the current swap curve, which represents the estimated annuitization cost. So, the swap curve liability ceiling would apply.
- For a GASB 45 postretirement medical plan transitioning from being unfunded to being funded, the credible funding policy would be a written policy that has been adhered to.
- For public plans, in the typical case, the funding discount rate solved for would appear on both sides of the equation (in both PVFC and PVFB). Beginning with a guess of the (arithmetic average) funding discount rate d_0 used for PVFC, PVFB would be solved for another discount rate d_1. PVFC is usually an increasing function of discount rate (a lower discount rate leads to accelerated funding, and a lower PVFC, and vice versa). (If PVFC is not an increasing function of the discount rate, investment de-risking is indicated.) PVAB is a decreasing function of the discount rate. So, the final discount rate solved for would lie between d_0 and d_1.

Importantly, because this approach reflects the correct fair value definition, it actually results in normative pension accounting rules. That is, plan sponsors are rewarded (via a higher discount rate) for 'doing the right thing.' For example, for a poorly funded plan with a risky investment allocation, additional contributions would lower PVFC more than dollar for dollar; the plan sponsor can increase his discount rate by accelerating the funding. In some cases a poorly funded plan might also lower PVFC by making the investment allocation riskier, but not beyond the FAS 87 universe average riskiness. For a well-funded plan, where the PVFC calculation is dominated by poor investment return scenarios, the plan sponsor can lower PVFC (and increase the discount rate) by de-risking the plan investments.

How does this model square with FAS 157? First, this model rejects paragraph B3b (essentially, the funding irrelevancy rule) as unsupported by valid economic theory. In determining a pension liability discount rate, rational choice demands that the investment opportunities and risks of an average pension plan sponsor be taken into account. Of the present value methods presented in Appendix B, this method is closest to Method 1 of the expected present value technique, using the weighted or unweighted risk-adjusted average plan sponsor contributions as the cash flows instead of benefit payments. And, the model includes a 'risk loading' to reflect the expectation that a typically risk-averse pension plan sponsor would be willing to pay an extra premium to transfer away pension risk. Additionally, the end product of this method is a risk-adjusted discount rate, which is then used in a traditional manner using arithmetic mean expected values for the other parameters to calculate the liability.

How does this model square with the European Financial Reporting Advisory Group's (EFRAG's) "The Financial Reporting of Pensions – A PAAinE Discussion Paper"? First, this paper disagrees that differences is defined benefit plan designs (such as cash balance plans) should lead to different rules. A proposed 'positive' definition of a defined benefit pension plan is a plan (other than a defined contribution plan) whose benefit liabilities can be calculated as projected future cash flows based on assumed parameters. The term benefit liabilities here encompasses aggregate benefit liabilities as well as benefit liabilities that have been attributed to time periods via an attribution method such as projected unit credit or traditional unit credit. This definition, when used along with the methodologies described in this paper, encompasses IASB's contribution-based promises, return-based promises, and higher-of promises.

Second, regarding liability measurement, this paper has two areas of disagreement. A risk-free discount rate is not appropriate, as discussed throughout this paper. Also, while significant assumptions should be disclosed, mere disclosure of arithmetic mean assumed parameters is not sufficient information for financial statement users. The assumed standard deviation of these parameters should also be disclosed, and the benefit liability should reflect this standard deviation. This paper agrees with EFRAG that expected expenses should be reflected in the benefit liability, and that the liability should not reflect the plan sponsor's credit risk (except for the risk of going out of business entirely in connection with future sponsor contributions to the plan.)

Importantly, there are some pension plan provisions (for example, an interest rate guarantee in a DROP) that are clearly impossible to value correctly without first doing stochastic forecasts (or, as with this model, the equivalent) to develop assumptions to use in a traditional deterministic model. It's time we realized that this is also the case with any funded defined benefit plan. The current method of getting a discount rate, whether it's using the expected return on assets or plugging in a risk-free rate, is analogous to calculating the present value of an immediate life annuity by using a certain annuity with the term set equal to life expectancy. It is naïve to expect to get the right answer that way.

Conclusion

This paper demands an immediate response from financial economists, and from actuaries and accountants who support the current financial economics viewpoint. First, they must explain why it is acceptable to apply the fallacy of sweeping generalization to arrive at the Law of One Price for Level 3 instruments. As they will be unable to do the impossible – that is, show how a non-arbitrageable instrument can be arbitraged – they must acknowledge this error and correct it immediately. Only then can rational accounting standards be created for pensions and other non-arbitrageable instruments.

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